Anisotropy in Weak and Strong Scintillation  
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I have put comments on these slides in the PDF file that will be available on-line (I expect).

Intensity scintillation is caused by density fluctuations in turbulent plasma, but compressive plasma turbulence is not well-understood.

Anisotropy in plasma turbulence is more poorly-understood.

So anything we can learn about anisotropy will be useful.

The effect of anisotropy is different in weak and strong scintillation.

Observationally:

- Scintillation in the ionosphere is very anisotropic.
- Anisotropy in the solar wind increases as the Sun is approached.
- Scintillation in the interstellar medium (IISM) has been thought to be quite anisotropic, but that might not be true.

The dynamics of turbulence are controlled by kinetic and magnetic energy density. The particle density is a “passive tracer” of the turbulence. So the spectrum of density in a compressive plasma is poorly understood.

The effect of anisotropy is different in weak and strong scintillation and what you can learn about the scattering plasma is different.

We know that scintillation in the ionosphere is very anisotropic although this has not been well-studied because the ionosphere is a complex, layered, and inhomogeneous medium.

Scintillation in the solar wind is also anisotropic and becomes much more anisotropic as the Sun is approached. The solar wind has been well-studied. In this case we have good evidence that the anisotropy is produced by a distinct wave mode (obliquely propagating Alfvén waves).

Scintillation in the interstellar plasma (ISM) often shows signs of anisotropy, and, until recently, it has been thought that very high axial ratio scattering was quite common. It is not at all obvious that IISM anisotropy would be similar to that of the solar wind, except that it is almost certainly magnetic field controlled and aligned.
**Strength of Scintillation**

Electron density fluctuations cause **angular scattering**, creating an **angular spectrum** of plane waves $B(\theta)$.

Intensity **scintillation** is caused by interference between components of this angular spectrum. Interference increases with distance from the scattering medium $L$.

**Weak scintillation** occurs near the scattering region where most of the angular spectrum is in-phase. The scintillation is due to interference of highly scattered waves with the **unscattered core**.

In **strong scintillation** further from the scattering region the unscattered core almost disappears. All waves interfere randomly with each other.

Between weak and strong scintillation **refraction** is important.

Scales $< R_F$ will scatter by **diffraction**, and scales $> R_F$ by **refraction**.

**Turbulence** has a spectrum of scales so diffractive and refractive scintillation will always occur, but their relative importance will depend on the distance.

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The phenomenon underlying scintillation is angular scattering. In small-angle forward-scattering we can think of angular scattering as causing an angular spectrum of plane waves. The spatial spectrum $E(K_x,K_y)$ is identified exactly with that angular spectrum as $K_x = k \sin(\Theta_x)$ and $K_y$ similarly.

Intensity fluctuations (scintillation) are caused by interference between components of this angular spectrum. Of course components will not interfere unless there is a phase difference between them. The phase difference $\phi$ increases with the scattering angle $\theta$ and the distance from the screen $L$ as $\phi = 0.5 \frac{\theta^2 L}{\lambda}$.

In weak scintillation most of the angular spectrum is in-phase, i.e. $|\phi| < 1$. We can define a Fresnel angle at which $|\phi| = 1$, i.e. $\theta_F = \sqrt{2 / k L}$. If most of the power in the angular spectrum is at $|\theta| < \theta_F$ then the scintillation is weak. In this case the scintillation is caused by the interference of highly scattered components with the unscattered core.

In strong scintillation the unscattered core shrinks to contain negligible power, all components are randomly phased and all interfere with each other. This is sometimes called “asymptotically strong scintillation” because there is an important region between weak and strong scintillation in which refractive effects are important.

By diffraction we mean where the scattering angle is directly related to the wavenumber of the irregularity through $K_x = k \sin(\Theta_x)$, like a diffraction grating. The diffractive angle is independent of the amplitude of the density irregularity. Whereas by refraction we mean the regime in which the scattering angle $k \Theta_x = \nabla_x (\text{Phase})$. The refractive angle depends linearly on the amplitude of the density fluctuation.
The strength of scattering is a distance effect.

The angular spectrum is ~ gaussian above ~ 0.1, below is a power-law tail.

Parabolic arcs are caused by this power-law tail.

- If the Fresnel angle is at the black circle the scintillation is weak whereas at the red circle the scintillation is strong. This is a distance effect.
- On the semi-log scale one can see that the angular spectrum looks gaussian above 10% of the peak brightness, below that one can see a power-law tail with exponent = -11/3.

- Parabolic arcs are caused by this power-law tail, which is why one needs high signal to noise ratio to observe them.

Bulk parameters like: spatial scale, bandwidth, scattering delay, can be modeled with a gaussian angular spectrum (or quadratic structure function), whereas parameters like parabolic arcs and the exponent of the scattered pulse tail depend on the non-gaussian region and thus the spectral exponent.
This effect is useful in the solar wind because it shows the direction of \( \mathbf{B} \).

In weak scintillation the intensity ACF has no ripples.

In weak scintillation it is easy to estimate the anisotropy if the major axis of the angular scattering is aligned with the velocity.

This has been very useful in the solar wind, where this alignment only occurs during coronal mass ejections (CMEs) and it is a reliable signature of a CME. Richard Fallows could show you many good examples.
Delay-Doppler Spectrum in Weak Scintillation

Each wave in the angular spectrum has a Delay and a Doppler shift. It interferes with the unscattered core, so we observe $T_D$ and $F_D$ of the scattered waves.

$$T_D = 0.5 (\theta_X^2 + \theta_Y^2) \frac{L}{c}$$
$$F_D = V_X \theta_X, \frac{1}{\lambda}$$

There is a 2D -> 2D mapping from $(\theta_X, \theta_Y)$ to $(T_D, F_D)$ with a 2 fold ambiguity in $\pm \theta_Y$.

We can recover $B(\theta_X, \theta_Y)$ or $P_{NE}(K_X, K_Y)$ from measurements of $S_{DD}(T_D, F_D)$.

The intensity of $S_{DD}(T_D, F_D)$ shows a slight dip centered on $F_D = 0$ this is due to AR.
Recognizing that weak scintillation is significant when the point-source, monochromatic characteristics are considered, a thorough analysis is provided. The secondary spectrum is the double Fourier transform of the joint probability density function of the intensity fluctuations at two different observing wavelengths. This allows a linearization of the problem, even though the root-mean-square intensity is much less than one. The solution is given by:

$$ S_2(f_i, f_i) = \frac{8\pi^2H_2}{V d_e \kappa_{yp}} \left[ P_{\phi} \left( \kappa_x = \frac{2\pi f_i}{V_{\perp}}, \kappa_{yp} \right) + P_{\phi} \left( \kappa_x = \frac{2\pi f_i}{V_{\perp}}, -\kappa_{yp} \right) \right] $$

where $\kappa_{yp} = \sqrt{8\pi^2|f_i|/d_e - (2\pi f_i/V_{\perp})^2}$, must be real.

There is a half-order singularity at $K_V = 0$ which forms a forward parabolic arc. The solution can be thought of as a transformation from $(f_i, f_i)$ to $(K_x, K_V) = k(\theta_x, \theta_V)$ and $1/K_V$ is the Jacobian of the transformation.

This analysis assumes weak scintillation and a narrow bandwidth, but the basic form is surprisingly robust. In strong scintillation the Jacobian becomes smoothed, but in a broader band and stronger scintillation the basic form of $S_2 \sim P_{\phi}(\sqrt{F_{\lambda}})$ along a parabola inside the Jacobian is maintained.
We can analyze the observed SDD by normalizing $F_D$ by $F_{DMAX}$. In this case all cuts through SDD at constant $T_D$ show the phase power spectrum. Likewise all cuts through SDD at constant $F_N$ show the axial ratio and its orientation.

In weak scintillation the SDD can be computed analytically and it has a very simple form. We can analyze the observations by normalizing $F_D$ by $F_{DMAX}$. In this case all cuts through SDD at constant $T_D$ look the same. These show the phase power spectrum. Likewise all cuts through SDD at constant $F_N$ look the same. These show the axial ratio and its orientation.

We can average over the entire SDD plane to improve the signal to noise ratio.
One can see that the cutoff due to the Jacobian remains very distinct up to AR = 8.
The power ratio in changing orientation by 90deg is large = AR(11/3).

The ability to measure the 2D phase spectrum provides an opportunity to test the hypothesis that the spectral exponent is different in parallel and perpendicular directions.
Sub-Summary (1)

In weak scintillation (for $AR \approx 8$) we have:
(1) the arc curvature;
(2) the axial ratio and its orientation;
(3) the 2-D power spectrum of electron density.

Few pulsar observations are in weak scintillation.

How far into strong scintillation is the weak scintillation approximation valid?
Corresponding cuts through $S_{\infty}(T_0, F_0)$ for $\text{Mb}^2 = 0.1$ (left) and 10 (right) \\
$\text{AR} = 1.5$ at 0 deg in both cases.

$\text{Mb}^2 < 10$ corresponds to $\nu^{1.5} > 64$ MHz at L band. \\
Power spectral exponent is easily measured. \\
Axial ratio and orientation can be measured. \\
Curvature of primary arc has much greater error than in weak scintillation.

So we can expect it to be quite straightforward to estimate the axial ratio and its orientation for pulsars with $\nu^{1.5} > 64$ MHz at L band when AR is small.

- In this case fitting the sharpness of the primary arc may well be the most precise measure of strength of scattering in this range of scintillation strength.

- At the AR gets larger $\text{Mb}^2$ becomes less useful. We adjust the turbulence level so the angular scattering on the major axis is the same as the isotropic case for $\text{Mb}^2 = 10$. 
What about greater AR?

At AR > 2 the strength of scintillation becomes ambiguous. One could be in weak scintillation in one direction but in strong scintillation in the orthogonal direction.

Here we have kept the rms scattering angle in the more highly scattered direction the same as it would be in an isotropic medium with the same Mb2. We have chosen Mb2 = 10 as the isotropic reference in the next two simulations for AR = 4 and AR = 10.

This keeps the bandwidth about the same, but a bit smaller. However the Mb2 is much larger and is no longer very meaningful.
The weak scintillation model is viable up to $AR = 4$. You can still find:
Power spectral exponent;
Axial ratio and orientation; and
Curvature of primary arc.
While observing these dynamic spectra one cannot miss the tilts in the “scintles”. As the scattering gets stronger tilts in the scintles in the dynamic spectrum become ubiquitous. You have probably all seen them and wondered about them. These are caused by the mean phase gradient over the scattering disc. The show up as an asymmetry in the parabolic arcs if the DD spectrum is taken over a region dominated by one tilt angle. However they also show clearly in the ACF even if the pulsar does not display parabolic arcs.

This is important because it is possible to measure the tilt of the ACF quite accurately and reconstruct the phase gradient in the direction of the velocity. Then by integrating the phase gradient one can reconstruct the DM along the path taken by the line of sight through the plasma. This has been proven with simulations and demonstrated for the pulsar J1603-7202 by Daniel R.
At AR = 10 it becomes difficult to fit the arc curvature independently unless the maximum scattering angle is within 30 deg of the velocity.

However in both cases the power spectrum is well-estimated. This is very interesting because we can search for different spectral exponents parallel and perpendicular to B (as proposed by Higdon).
Sub-Summary (2)

The weak scintillation model for $S_{\text{DD}}(P_d, F_d)$ is useful up to strengths equivalent to $M_b^2 = 10$ for an isotropic medium.

It can be used at that level of scattering (as measured with the bandwidth) for $A_R < 10$.

This covers a lot more pulsar observations, including the beautiful results from J0437-4715 that will be shown by Daniel Reardon.

Note that in this range of scintillation strength $M_b^2 < 10$, it is very difficult to measure the basic time and frequency scales of the dynamic spectrum because there is often only a few scintles in the dynamic spectrum. However it is easy to see parabolic arcs because they are caused by smaller scales structures in both time and frequency. This also means that attempts to estimate the time and frequency scales will always underestimate them both (when $M_b^2 < 10$).
**Why is the curvature of the forward arc important?**

It allows us to apply the scintillation time scale technique proposed and tested by Andrew Lyne and colleagues and first applied precisely to the relativistic binary J1141-6545 by Steve Ord and colleagues. The idea is that the time scale of scintillations is proportional to the velocity. The constant of proportionality can be determined if the pulsar is a binary and the orbital velocity is known. Once the velocity is calibrated the orbital inclination can be determined (and the location of the scattering medium).

The did not work well until Ord et al applied it to J1141-6545. It also works well on the double pulsar J0737-3039. The reason it works well on those pulsars is that they are very short period binaries - the binary orbit lies entirely within the scattering disc. Then the scintillation parameters are well-behaved over the orbit. However they are not well-behaved on longer spatial scales because the level of turbulence changes on an AU spatial scale. Furthermore the time scale depends on the anisotropy so two additional parameters must be fit. This makes the solution very difficult - but just possible.

With longer period binaries the spatial scale varies over the orbit and good fits are much harder to obtain. However the curvature of the primary arc is not affected by the level of turbulence or the anisotropy. So the fit to the orbital velocity and the Earth’s velocity is very much more precise. And the technique can be extended to any pulsar for which the primary arc can be detected. In particular it works beautifully with J0437-4715 as Daniel will show.
In summary:

(1) the curvature of the parabolic arc is extremely valuable and we are learning how to best deal with it.

(2) The estimate of the 2-D phase power spectrum remains robust regardless of the axial ratio or orientation thereof. And it remains good up to substantial scintillation strength. We have not begun to take advantage of this. We should search for deviations from the simple Kolmogorov behavior in the 2-D power spectrum.

(3) It will be possible to fit the $S_00(T_0, F_0)$ to obtain estimates of AR and orientation but this will become more difficult at higher axial ratios and stronger scintillation.

What else is interesting?
2. How to best estimate $S_{DD}$?

We start from the dynamic spectrum $dI(f, t)$. The Doppler frequency is obtained by a Fourier transform (FT) along the time axis. The Delay is obtained by a FT along the frequency axis. So $S_{DD}(T_D, F_D) = |\text{FT2}(dI(f, t))|^2$.

1. **Minimize the RFI:** To first order RFI appears as broad band bursts or narrow band long duration noise, i.e. as vertical or horizontal lines. These will appear along the axes of the FT2, so they won’t directly interfere with measurements of a parabolic arc. However they do leak away from the axes and this is a significant source of measurement error, so editing RFI in $dI(f, t)$ is even more important than we have realized before.

2. **Interpolate $dI(f, t)$** to an equally spaced grid in $dI(\lambda, t)$. If the fractional bandwidth $df/f < 33\%$ this can be done with a simple interpolation. If $df/f > 33\%$ then a least squares interpolation will be required. Even with the relatively narrow bandwidth of the Parkes L-band receiver this improves the arc curvature accuracy significantly.

3. **Prewhiten $dI(\lambda, t)$** in 2D to minimize spectral leakage (and post-darken afterwards). First differencing is normally adequate.

4. **Consider using a spectral window** to sharpen the edge response. Hamming and Bartlett have been used successfully depending on the signal to noise ratio and the sampling of the dynamic spectrum.
The New ultra-broad band receivers are going to be very useful.

Simulation of Dynamic Spectrum on Parkes UWL Receiver

The New ultra-broad band receivers are going to be very useful.

4000 channels each 83.6 MHz wide for a 6:1 frequency range

For a pulsar at 500 pc with a velocity of 100 km/s the time scale for \( r_f \) is about 4 hrs. So the x-axis corresponds to 13 days

The mean intensity is unity, so we occasionally see spikes something 16 times the mean. You can have a lot of fun with this.

We haven't analyzed simulations of these carefully because we are just starting to get the necessary data
Some obvious analyses of UWL

You treat it like a bunch of narrow band observations. Intensity variance vs frequency, diffractive spatial scale vs frequency, diffractive electric field scale vs frequency. The electric field spatial scale is not trivial to estimate because there are various refractive effects also present.

The spatial scale at 600 MHz ~ 24 min. At 2.3 GHz it is 2 hrs. However we often only have 1 hr of observations so we will underestimate the spatial scale.

Similarly we couldn’t measure the pulse width when it is < 80 ns, but we could measure the coherence bandwidth. However when the frequency gets above 1.4 GHz the coherence bandwidth is limited by the receiver bandwidth. Our bandwidth estimate will be underestimated.

The intensity variance is OK in weak scintillation and also in very strong scintillation, but it jumps around drastically when refraction is important. This is because we don’t have many refractive spatial scales in the observing window, but we have very many diffractive scales.

You can also break it into 83 MHz sub channels and compute the pulse width and frequency bandwidth in each sub channel. They only agree with strong scattering theory at the lowest three sub channels.
There is very interesting large scale structure in here that we have not tried to understand yet.
Dynamic Spectra of Parkes UWL

top: equal frequency steps;

It is interesting that we see a filled forward arc over the whole band, which includes quite strong scintillation at the lowest frequencies.

bottom: equal wavelength steps

We need to do this simulation with anisotropy and see if we can obtain a filled arc at high frequencies and a forest of reverse sub-arcs at high frequencies. My guess is no. I think that we’ll also need to introduce inhomogeneities on a spatial scale ~ the refractive scale. But that’s more of a guess than anything.
Finally: Parkes UWL is not the only broad-band receiver!
The new ultra-broad-band receivers allow for great improvement in scintillometry.
We already see curious changes in flux over the Parkes UWL band.
We can see interesting refractive effects in the simulations.
We see strong refraction in Kaira ionospheric scintillations (Fallows et al, 2014).
Refraction has not been studied much, it may be open to some new ideas.

The ionospheric velocity is not constant and linear. It often reverses direction around midnight. The right hand spectrum may reflect a velocity reversal.

Velocity reversals also occur in many binary pulsars and many pulsars with low proper-motion. In analyzing the time variation of any kind of scintillation we need to keep in mind that the trajectory of the LOS through the medium is not necessarily linear.